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RESEARCH ARTICLE

Problem-Posing Skills and Thinking Styles of Preservice Teachers

Pınar GÜNER 

Department of Mathematics Education, Hasan Ali Yücel Faculty of Education, İstanbul University-Cerrahpaşa, İstanbul, Turkey

Abstract

The purpose of this study was to investigate the problem-posing skills of preservice teachers and to examine the relationship between their problem-posing skills, thinking styles, and academic achievement. A total of 32 senior preservice middle school mathematics teachers posed a problem, and they completed the thinking style inventory. A rubric for evaluation of problem-posing skills was used to analyze the problems with a qualitative approach. It was found that the problem-posing skills of preservice teachers were high, but they were set to pose routine problems. It was found that there was no significant relationship not only between problem-posing skills and thinking styles but also between academic achievement and problem-posing skills of preservice teachers. The findings also showed that academic achievement was significantly, positively, and moderately related to executive, hierarchical, and internal thinking styles, whereas it was significantly, negatively, and moderately related to external thinking style.

Keywords: Problem-posing, thinking styles, academic achievement, preservice teachers, mathematics, teacher education

Introduction

Lately, problem-posing, which is one of the core concepts of mathematics, has become important. It is accepted as an important approach that can improve the learning and teaching of mathematics (Cai et al., 2015; Singer et al., 2015). Problem-posing requires an understanding of the structure of the mathematical problem. However, there are many misconceptions about the meaning and structure of a mathematical problem (McDonald & Smith, 2020). A mathematical problem is a task (a) in which the student is interested and engaged and for which he wishes to obtain a resolution; and (b) for which the student does not have a readily accessible means by which to achieve that resolution” (Schoenfeld, 1989, p. 87-88). It is separated into two topics as routine and nonroutine problems. Routine problems refer to the daily life situations with which students are familiar and make solutions with four basic operations. Nonroutine problems reflect the situations that students are not familiar with and require more complex skills such as exploring patterns, reasoning, and using various strategies (Altun, 2014). Accordingly, problem-posing is defined as the process through which students interpret concrete situations and form meaningful (i.e., nontrivial) mathematical problems

Corresponding Author: Pınar GÜNER E-mail: pinar.guner@istanbul.edu.tr

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with their mathematical experience and personal perspective (Stoyanova & Ellerton, 1996). In contrast, Silver (1994) defined the problem-posing as both forming new problems and reformulating the existing problems. Problem-posing also requires the use of problem-solving abilities (Cai & Hwang, 2002). However, it is not generally understood that problem-posing is an integral part of problem solving.

Problem-posing fosters students to understand conceptually, think logically, and communicate mathematically (National Council of Teachers of Mathematics [NCTM], 1991). Research has pointed out the positive effects of problem-posing on students with the support to reason, investigate, and use appropriate mathematics (Silver & Cai, 1996; Toluk-Ucar, 2009). Some studies also emphasized the relationship between problem-posing and mathematics achievement as well as problem solving (Christou et al, 2005; Ellerton, 1986). Moreover, its contribution includes the development of critical thinking (Bonotto, 2013), mathematical ability (Silver, 2013), and reading comprehension (Cai et al., 2013). It also improves content knowledge, problem-solving, and high-order thinking skills and beliefs (Chang et al., 2012; Kaberman & Dori, 2009; Toluk-Ucar, 2009). In school, mathematics students have various experiences related to problem solving. However, the attention of problem-posing is neglected generally. All students should try to create their own problems (Kilpatrick, 1987) because people are not considered to have fully experienced mathematics without solving the problems they have created (Polya, 1957). This activity provides information for teachers to learn more about their students' conceptual understanding, the skills of problem solving, and creativity (Cai & Hwang, 2002; Ellerton, 1986; Kilpatrick, 1987; Silver & Cai, 1996). It reveals the need for more attention to problem-posing.

One of the important concepts in education is the thinking styles, because no action is independent of thinking. Thinking styles vary with the interaction between individuals and the environment (Sternberg, 1997). Thinking styles are defined as the preferred ways of using the ability one has (Sternberg & Grigorenko, 1997). Research highlights that the focus on thinking styles can help to improve teaching and learning (Grigorenko & Sternberg, 1997; Sternberg, 1997). According to Zhang (2003), thinking styles provide to gain insight into how an individual learns and uses information. It points to the importance of knowing the thinking styles of both students and teachers. The results of the studies related to thinking styles mainly revealed that students' thinking styles are affected by their personal characteristics and learning environments. The researchers emphasized that the consistency between the thinking styles of students and those of the teachers are related to students' academic achievement, and they referred to the contribution of students' thinking styles on their achievement (Grigorenko & Sternberg, 1997; Sternberg

& Grigorenko, 1995; Zhang & Sternberg, 1998). Moreover, the differentiation of the reactions people give to the problems they have encountered and the solutions they find for the problems points to different thinking styles (Çatalbaş, 2006). It is believed that there may be an interactive relationship between preservice teachers' thinking styles, the problems that are posed by them, and their academic achievements.

Literature Review

Problem-posing

In many countries such as China and the United States, the importance of problem-posing is emphasized in mathematical curricula (Singer et al., 2015). There has been a tendency to integrate problem-posing into mathematics instruction at different class levels in schools recently (Cai et al., 2015). In Turkey, the Ministry of National Education [MoNE] (2018) also emphasizes that problem-posing studies in mathematics teaching programs be included in every mathematics subject from the first grade. Problem-posing is important in terms of both teachers and students, because the problems posed by teachers have an impact on the students' learning of mathematics and the achievement of goals in mathematics teaching (NCTM, 2000). It also provides to learn more about the thinking and understanding ways of students (Leung, 2013). Despite the importance of problem-posing in mathematics education, students, teachers, or educators have paid little attention to it (Kilpatrick, 1987; Silver, 2013). In general, teachers tend to skip the problem-posing in lessons and do not allow students to pose mathematical problems (Lee, 2020). According to NCTM (1991), every student should have the opportunities to state and pose their own problems. One of the teachers' goals should be to educate students as good problem posers (Cai et al., 2015; Crespo, 2003). For effective mathematics teaching and learning, teachers should be capable of determining and posing appropriate problems and tasks to improve the mathematical thinking and understanding of students by making them active (Kulm, 1994).

To gain students' problem-posing skills, teachers must have these skills and be able to use them firstly (Li et al., 2020). Although some research has stated the capability of students and teachers in posing mathematics problems (Cai et al., 2013; Cai & Hwang, 2002; Crespo, 2003; Kar, 2015; Stickles, 2011), others have pointed out the difficulties in the process of problem-posing and ensuring the validity and quality of problems (Cai & Hwang, 2002; Crespo & Sinclair, 2008; Osana & Royea, 2011). According to their research, which investigated the preservice and in-service teachers' perspectives on problem-posing, Hospesova and Ticha (2015) believe that problem-posing is important but difficult, that problem solving is easier than prob-

lem-posing, and that teachers do not have to pose problems, but the problems posed by teachers are more meaningful and helpful for students and their comprehension. Crespo and Sinclair (2008) explained that the reason for preservice teachers' difficulties in posing problems is not being familiar with this activity. Because problem-posing is an important activity in school mathematics, this reveals the need to investigate the problem-posing skills of preservice teachers to determine the lack of them and help to improve these skills, so that their contributions to students may increase in mathematics classes. This research can be useful in terms of providing information for teacher training programs and the professional development of preservice teachers.

Thinking styles

The theory of mental self-government (Sternberg, 1997) refers to thinking styles that are used by people in various contexts, such as in school, university, home, work, and community (Zhang, 2001a). Thinking styles do not mean abilities, instead they refer to the ways people use their abilities (Sternberg, 1997). From carrying out everyday activities to making a decision, thinking is necessary, and the thinking styles of people may change with the demands of different environments and in time. According to the theory of mental self-government (Table 1), there are 13 thinking styles under five dimensions, including functions, forms, levels, scopes, and leanings (Sternberg, 1997).

Table 1
Descriptions of Thinking Styles in the Theory of Self-Government

Thinking Styles	Descriptions	
Functions	Legislative	It requires using creative strategies and choosing one's own activities.
	Executive	It refers to follow instructions and implement tasks with set guidelines.
	Judicial	It includes the preferences to evaluate the performance or product of one's own and other people.
Forms	Monarchic	It is concerned with focusing on only one goal at a time.
	Hierarchical	It addresses the preferences to work on several prioritized tasks and distribute attention to multiple goals at once.
	Oligarchic	It refers to engage in multiple tasks within the same time without considering priorities.
	Anarchic	It includes working on flexible tasks in terms of what, where, when, and how to do.
Levels	Global	Its style requires focusing on the whole picture and abstract ideas related to an issue.
	Local	It addresses the preferences to engage in concrete details of tasks.
Scopes	Internal	It is concerned with working on tasks by himself or herself.
	External	It refers to work on tasks with other people collaboratively.
Leanings	Liberal	It addresses the preferences to use new ways and work on tasks that include novelty, originality, and ambiguity.
	Conservative	It is concerned with following the existing rules and procedures in tasks and looking for conformity.

The research related to thinking styles has focused on the relationship between thinking styles and various concepts, such as achievement (Cano-García & Hughes, 2000; Grigorenko & Sternberg, 1997; Zhang, 2002b; Zhang & Sternberg, 1998), learning approach (Zhang & Sternberg, 2000), teaching styles (Zhang, 2008), students' socioeconomic status (Sternberg & Grigorenko, 1995), and personality trait (Zhang, 2002a, Zhang, 2002b). The studies also emphasize the relationship between the thinking and teaching styles of teachers and the relationship between thinking styles and learning approaches (Zhang, 2000; Zhang & Sternberg, 2000). Zhang (2004) also indicated that university students with different thinking styles had different teaching approaches regardless of age, gender, university class level, and academic discipline. There were studies that revealed the most and least preferred thinking styles by teachers (Duman & Çelik, 2011; Özbaş & Sağır, 2014; Yu & Zhu, 2011; Zhang, 2008) and preservice teachers (Çubukçu, 2004; Uyanık, 2017). However, the scarcity of studies examining the relationship between thinking styles and problem-posing skills draws attention. It is known that from mathematicians to primary school students (Gray & Pitta-Pantazi, 2006), all individuals have thinking preferences while solving mathematics (Moutsios-Rentzos & Simpson, 2010). Owing to the differences in the nature of problem situations, it is believed that the thinking styles of students may affect their problem-posing skills. Zhu (2011) also addresses that the learning and teaching environment are not independent of the thinking styles and personal characteristics of students and teachers. At this point, identifying problem-posing skills of preservice mathematics teachers who will be future mathematics teachers and establishing how these thinking styles are related to their tendency in posing problems and academic achievement can help educators to improve instruction and assessment and to provide some guidance for better performance.

This study was aimed to investigate the problem-posing skills of preservice teachers and the relationship between their problem-posing skills, thinking styles, and academic achievement. For this purpose, the following research questions were investigated:

- What is the level of the problem-posing skills of preservice middle school mathematics teachers?
- Is there any relationship between the problem-posing skills and the thinking styles of preservice middle school mathematics teachers?
- Is there any relationship between the problem-posing skills and the academic achievement of preservice middle school mathematics teachers?

- Is there any relationship between the thinking styles and the academic achievement of preservice middle school mathematics teachers?

Method

In this study, the convergent parallel mixed model was utilized that includes the collection of qualitative and quantitative data concurrently and the analysis and comparison of the collected data separately (Creswell, 2013). Qualitative data were obtained by the analysis of the problems that were posed by preservice teachers, whereas quantitative data consisted of the scores obtained from thinking styles inventory and according to problem-posing rubric.

Participants

This research was conducted with preservice teachers as a component of the problem-solving course. The problem-solving course took place in the last semester of a 4-year undergraduate mathematics teacher education curriculum in a university in the northwest part of Turkey. Within the context of this course, preservice teachers were informed about the concepts such as problem; mathematical problem; the types of problems such as routine and nonroutine problems, problems with multiple solutions, unsolved problems, problems with missing data, and so on; the steps of problem solving; the strategies that are needed for problem solving; and the preservice teachers solved dominantly nonroutine problems with the guidance of the researcher who is also the lecturer of the course during the 12 weeks. Thus, they gain deep insight into mathematical problems and problem solving. However, they did not have experience in posing problems in this course. This study was carried out with 32 preservice middle school mathematics teachers (27 females and 5 males) in their senior year. The grade point average (GPA) of the preservice teachers varied between 2.50 and 3.65 out of 4.00. They had already completed almost all of their compulsory courses on teaching pedagogy.

Data Collection

To determine the problem-posing skills of preservice teachers, the participants were asked to pose a problem at the end of the problem-solving course. There was no limitation in terms of mathematical subjects, operations, contexts, situations, and difficulty for the problems that they would pose. They were asked to pose a problem and solve them according to problem-solving steps as they did in the lessons. At 1 week after the end of the course, the problem statements and the solutions were collected. At this time, the preservice teachers filled the thinking styles inventory developed by Sternberg and Wagner (1992), which was adapted to Turkish by Fer (2005).

According to the reliability and validity studies conducted by Fer (2005), the results of factor analysis for the construct validity of the inventory addressed 13 subscales under the five dimensions as in the original inventory. The total internal consistency reliability of the inventory was found to be .90. The subscales had internal consistency, and positive and significant values were found at .01 level in all subscales (Fer, 2005). For this study, the internal consistency Cronbach alpha's coefficient was found to be .90. The inventory includes 104 items that address 5 dimensions and 13 subscales that each has eight items. The inventory is a 7-point Likert scale ranging from 1 to 7 (in which 1 indicates not well and 7 indicates extremely well). The thinking styles inventory does not have a total score because a thinking style that is dominant for an individual is measured independently from the other subscales. As the score increases, it is accepted that the thinking style is at a high level (Fer, 2005). The thinking styles inventory was completed by preservice teachers in approximately 30–45 minutes. Preservice teachers' GPAs throughout undergraduate education, available from participant records, served as a measure of their academic achievement.

Data Analysis

The scoring rubric of problem-posing skills developed by Özgen et al. (2017) was used to evaluate the problems posed by preservice teachers. The criteria of the rubric are based on literature in terms of the properties that mathematical problems need to have, such as mathematical language (Gonzales, 1994), grammar rules suitability (Gonzales, 1994), solvability (Silver & Cai, 1996), originality (Chang et al., 2012; Gonzales, 1994), and quality and quantity (Chang et al, 2012). The rubric includes seven criteria that are scored at four levels. The minimum score is 0, and the maximum score is 3 for each criterion. The range of levels was classified as Level 1 for values between .00 and .75, Level 2 for values between .76 and 1.50, Level 3 for values between 1.51 and 2.25, and Level 4 for values between 2.26 and 3.00. To ensure the reliability of the scoring procedure, the problems of the preservice teachers were scored independently by two mathematics education experts, one of whom was the researcher of this study. The interrater reliability was found to be about 84%. To resolve the disagreements on the scores, both educators discussed differences to reach a consensus. Moreover, to answer the research questions of the study, the following statistical procedures were conducted with the Statistical Package of the Social Sciences program. Descriptive statistics are presented to reveal the problem-posing skills of preservice mathematics teachers. Moreover, the correlation coefficients were calculated for the scores of problem-posing skills,

thinking styles, and GPA to test whether there was any relationship between them.

Results

The results are presented in two parts: qualitative and quantitative. The quantitative part includes the descriptive statistics and correlation analyses results. The qualitative part involves the examples of problems that were posed by the preservice teachers.

Statistical Analysis

To determine the problem-posing skills of the preservice teachers, the mean and standard deviation (SD) values were calculated and presented in Table 2.

Table 2
The Rubric Statistics for Problem-Posing Skills: Ms and SDs

Problem-posing criterion (n = 32)	M	SD
Using the language of mathematics	2.28	.77
Grammar and expression suitability	2.15	.95
Suitability to acquisitions	2.40	.97
Quality and quantity of data	2.50	.80
Solvability	2.56	.87
Originality	1.31	.59
Solving the problem posed by the student	2.81	.59
Rubric scores	2.29	.55

Note. M = mean; SD = standard deviation.

Table 2 shows that according to the general rubric scores (2.29), the average of problem-posing skills of the preservice teachers were found at Level 4. It reveals that they are successful in posing their own problems. In terms of subcriteria analysis, it is seen that the preservice teachers' skills of using the language of mathematics (2.28) and solving the problems posed by themselves (2.81) were at Level 4. Moreover, the scores of their problems' suitability to acquisitions (2.40), quality and quantity of data (2.50), and solvability (2.56) were at Level 4. It indicates that preservice teachers have the tendency to use appropriate mathematical language and data and consider whether the problems are suitable and solvable while posing the problems. In contrast, the level for grammar and expression suitability criterion (2.15) was found at Level 3. It refers to some grammatical mistakes of the preservice teachers as posing problems. The originality of the problems (1.31) was found at Level 2. It points out that the preservice teachers are not successful in posing original problems. The results show that preservice teachers are mostly able to solve the problems that they posed. However, there were also preservice teachers who could not solve the

problems they created or created problems that could not be solved, even if it was a small amount.

Table 3 contains the summary statistics from the correlation analyses between GPA and the scores of problem-posing rubric and thinking styles inventory.

Table 3

Pearson Correlation Coefficients Between GPA, Problem-Posing Skills, and Thinking Styles

Variables	GPA	Problem-Posing Skills
	Correlation Coefficient (r)	Correlation Coefficient (r)
Problem-posing skills	-.148	1
Legislative	.129	.235
Executive	.382*	.004
Judicial	.031	.074
Monarchic	.225	.246
Hierarchical	.376*	-.017
Oligarchic	.162	.062
Anarchic	-.108	-.136
Global	.047	.014
Local	.324	.165
Internal	.475**	.131
External	-.446*	.108
Liberal	-.058	-.134
Conservative	.318	-.050

Note. * $p < .05$, ** $p < .01$, GPA = grade point average.

In Table 3, it is seen that there was no significant relationship between problem-posing skills and thinking styles of the preservice teachers ($p > .05$ for all thinking styles). Furthermore, there was no significant relationship between academic achievement and problem-posing skills of the preservice teachers. The findings also show that academic achievement was significantly, positively, and moderately related to executive ($r = .382, p < .05$), hierarchical ($r = .376, p < .05$), and internal ($r = .475, p < .01$) thinking styles, whereas it was significantly, negatively, and moderately related to external ($r = -0.446, p < .05$) thinking style. These findings show that the preferences that are concerned with following instructions, implementing guidelines, taking priority, considering hierarchy, and working on tasks on one's own may contribute to the academic achievement of preservice teachers. The preference for working on tasks collaboratively may have negative effects on academic achievement.

Problems Posed by Preservice Teachers

To evaluate the problem-posing skills of the preservice teachers, the problems that they posed were analyzed qualitatively using the scoring rubric of problem-posing and the examples are presented. In the first example (Figure 1), it is understood that the preservice teacher tried to pose a problem on the basis of going backward strategy. It is seen that the problem was easy and routine.

Problem

Bir masanın üzerinde belli bir sayıda kitap var. 4 kişi sırayla gelerek masanın üzerindeki kitapların yarısını alıyor yarısını bırakıyor. Sonraki gelen kalan kitapların yarısını alıp yarısını bırakıyor. 4. kişiden sonra masanın üzerinde hiç kitap kalmıyor. Başlangıçta masada kaç kitap vardır?

There are a certain number of books on a table. 4 people come in turn and take half of the books on the table and leave the half. The next person takes half of the remaining books and leaves the half. After the 4th person, there are no books on the table. How many books are on the table at the beginning?

Solution

	Alınan	Kalan
4. kişi.	1	0
3. kişi	1	1
2. kişi	2	2
1. kişi	4	4
	<hr/>	
	8	

Figure 1.


The First Example for Level 1

Here, it is seen that the context of problem is not suitable for solution. If the third person takes 1 book and leaves 1 book on the table, the fourth person will not be able to take half of 1 book. Thus, the mathematical set-up of the problem is not correct. However, it is seen that the preservice teacher took 1 book, considering it as a half, and wrote 0 as the remaining. The use of mathematical language, grammar, expressions, directions, and data was inappropriate. Moreover, the problem is an ordinary type, and it is problematic in terms of solvability. Thus, when the scores for all the criteria were considered, the problem-posing skills of the preservice teacher were evaluated as Level 1.

In the second example (Figure 2), it is seen that the preservice teacher posed a problem, including some rules and directions. She asked a problem that students had to consider the various possibilities, using a term such as the maximum value.

Here, it is understood that the preservice teacher wanted to state that the product of the numbers in the circles on the right and left sides of the hexagon is equal to each other and equal to the section marked A. However, it is seen that she only mentioned the equality of the right side to A in the problem and the comprehen-

Problem



- Yukarıdaki çemberlerin her birinin içinde farklı birer rakam bulunmaktadır.
- Altgenin sağındaki rakamların çarpımı birbirine eşit ve A değerine sahiptir.

Buna göre, A'nın alabileceği en büyük değer kaçtır?

- There is a different number in each of the above circle.
- The product of the digits to the right of the hexagon is equal to each other and has a value of A. What is the maximum value that A can get?

Solution

9 (3X3)	8 (4X2)	1	A	6 (3X2)	3	4
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Figure 2.

The First Example for Level 2

sibility of the problem is low. This problem is an example of inappropriate use of mathematical language, grammar, and expression. Moreover, the suitability to acquisitions and the quality and quantity of data were evaluated to be weak because the directions and data are not enough for solving the problem. On the contrary, although it cannot be solved owing to the lack of data and expression, it is seen that the preservice teacher solved it by completing the missing data and directions in the solution. The problem was evaluated to be ordinary. In general, this preservice teacher's problem-posing skills were determined as Level 2 for this problem.

In the third example (Figure 3), the preservice teacher created a problem related to the probability that contained many instructions and rules that should be followed carefully. It is seen that she tried to pose a problem that was not easy and not routine.

Although this problem was found appropriate in terms of the suitability of grammar and expressions, some deficiencies in the use of mathematical language, instructions, and data drew attention. It is seen that the preservice teacher did not state the number of faces on the dice and how to write the powers of 2 on it. She writes, "The powers of 2 are written on the faces of the dice"; however, which powers? or respectively? It is difficult for students to understand that there will be on the faces. In contrast, the expression "Everyone will take one throw" is confusing. It is believed that she wanted to say that each throw will be made in turn. It is seen that the preservice teacher solved it by completing the missing data and directions. The problem

Problem

Rabia ve Erva bir zar ile oyun oynuyorlar. Zarın yüzeylerinde ikinin kuvvetleri yazıyor. Oyunun kuralını şöyle belirliyorlar:

- Eğer zar atıldığında gelen sayı tam kare ise, atan kişi 5 puan kazanacak.
- Eğer zar atıldığında gelen sayı tam kare değil ise, atan kişi 3 puan kaybedecek.
- Herkes bir atış yapacak

Oyuna ilk Rabia başladığına göre, 6 atış yapıldığında Rabia'nın Erva'yı yenme olasılığı kaçtır?

Rabia and Erva play with a dice. The powers of two are written on the faces of the dice. They determine the rule of the game as follows:

- If the number that comes when dice is rolled is a perfect square, the person who rolls the dice will get 5 points.
- If the number that comes when dice is rolled is not a perfect square, the person who rolls the dice will lose 3 points.
- Everyone will take one throw.

If Rabia starts the game firstly, what is the probability of Rabia defeating Erva after 6 throws?

Solution

1 atış için 2 durum vardır.

R:55	E:50
R:47	E:50

1 durum için Rabia önde, 1 durum için Erva önde

2 atış için 4 durum vardır.

R:55	E:55
R:55	E:47
R:47	E:55
R:47	E:47

1 durum için Rabia önde, 2 durum için berabere, 1 durum için Erva önde

3 atış için 8 durum vardır.

R:55	E:55	R:60
R:55	E:55	R:52
R:55	E:47	R:60
R:55	E:47	R:52
R:47	E:55	R:52
R:47	E:55	R:45
R:47	E:47	R:52
R:47	E:47	R:45

4 durum için Rabia önde, 4 durum için Erva önde

4 atış için olan durum da benzer şekilde listelenirse:

5 durum için Rabia önde, 6 durum için berabere, 5 durum için Erva önde

Koyu renklerle yazdığımız sayı ilişkileri, bidede Pascal üçgenini hatırlatır.

				1										
					1									
						1								
							1							
								1						
									1					
										1				
											1			
												1		
													1	
														1

Ortadaki değerler berabere kalma durumunu, sol ve sağdaki değerler toplamı da Rabia ya da Erva'nın önde olma durumunu göstermektedir.

Buna göre 6 atış yapıldığında 22 durum için Rabia önde, 20 durum için berabere, 22 durum için Erva önde olur.

Yani Rabia'nın Erva'yı yenme ihtimali $\frac{22}{64}$ bulunur.

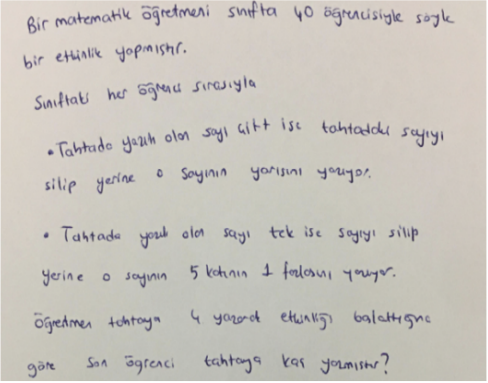
Figure 3.
The First Example for Level 3

was evaluated to be partly original because it can be distinguished from the classical problem type. In general, the problem-posing skills of this preservice teacher were accepted at Level 3.

In the fourth example (Figure 4), the problem posed by the preservice teacher includes 2 different instructions on the basis of simple arithmetic operations. It is seen that it is a problem that students are used to.

This problem was found appropriate and adequate in terms of mathematical language, grammar, suitability of expression, directions, data, and solvability. It is seen

Problem



Bir matematik öğretmeni sınıfta 40 öğrencisiyle şöyle bir etkinlik yapmıştır.

Sınıftaki her öğrenci sırasıyla

- Tahtada yazık olan sayı çift ise tahtadaki sayıyı silip yerine o sayının yarısını yazar.
- Tahtada yazık olan sayı tek ise sayıyı silip yerine o sayının 5 katının 1 fazlasını yazar.

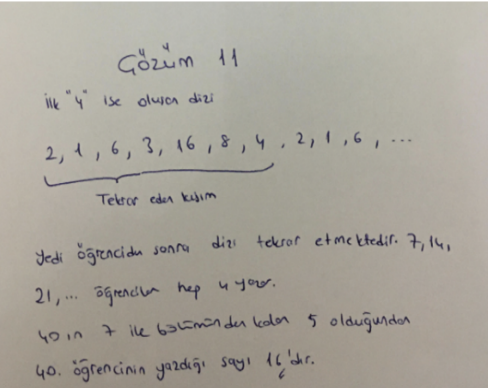
Öğretmen tahtaya 4 yazarak etkinliği başlattığına göre son öğrenci tahtaya kaç yazmıştır?

A mathematics teacher made an activity with 40 students in the class. Each student in the class followed the below directions respectively,

- If the number written on the board is even, he deletes the number and writes the half of that number on the board.
- If the number written on the board is odd, he deletes the number and write 1 more 5 times that number on the board.

If the teacher started the activity by writing 4 on the board, which number did the last student write on the board?

Solution



Çözüm 11

İlk "4" ise oluşan dizi

2, 1, 6, 3, 16, 8, 4, 2, 1, 6, ...

Tekrar eden kısım

7'den itibaren sonraki dizi tekrar etmektedir. 7, 14, 21, ... öğrenciler hep 4 yazar.

40'ın 7 ile bölümünden kalan 5 olduğundan 40. öğrencinin yazdığı sayı 16'dır.

Figure 4.

The First Example for Level 4

that she solved the problem by pointing out the pattern easily. However, the originality of it was low, and its type was a routine problem. The problem-posing skills of this preservice teacher were evaluated as Level 4.

In the fifth example (Figure 5), it is seen that the preservice teacher tried to pose a problem on the basis of a general rule about patterns. She explained what kind of relationship was in the pattern.

Problem

Bir üzüm motifi yapmak için elimizde açık yeşil ve koyu yeşil renk ipler vardır. Başlangıçtaki zincir koyu yeşil ile şekilip ardından bir koyu yeşil zincir bir de açık yeşil zincir şekillenir. Her basamakta koyu yeşil zincirden iki koyu yeşil zincir; açık yeşil zincirden bir açık yeşil zincir atılacak şekilde motif devam ederse 10. sıradaki ne kadar açık yeşil ne kadar koyu yeşil zincir olur?

We have light green and dark green ropes to make a grape motif. Initially dark green, then a dark green shape and a light green shape are placed. If the motif continues like that two dark green shapes and one light green shape are placed in each step, how much light green and dark green shapes will be in the 10th row?

Solution

Elde ettiğimiz bağıntıyı 10. sıra için hesapladığımızda $2^{10} + 1$ tane zincir olacaktır ve bunlardan yalnız 1'i açık yeşildir.

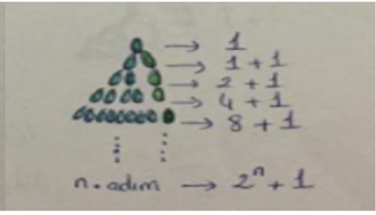


Figure 5.

The Second Example for Level 1

For this problem, it is difficult to understand the information the preservice teacher gave and what she wanted to ask. The instructions given in the problem and the shape she drew in the solution are not consistent with each other. It is seen that her solution is not correct for this problem. She states that “The motif continues like that two dark green shapes and one light green shape are placed in each step”; however, how she reached the general term is not clear. It can be said that the preservice teacher could not pose an appropriate and adequate problem in terms of mathematical language, grammar, directions, and data. The problem is not solvable and meaningful. When all criteria were considered, the problem-posing skills of the preservice teacher were determined as Level 1.

In the sixth example (Figure 6), it is understood that the preservice teacher tried to pose a problem with rounding to tens and hundreds.

It draws attention that the problem is quite long, and the preservice teacher could not express the problem clearly and briefly. Mathematical concepts were confusing, and there was incoherency. For example, the preservice teacher expressed rounding to 10 and 100 as predicting, but there is no prediction here. The statement that “I will subtract numbers” is not understandable. She should have expressed that “I will

<p>Problem</p> <p>Ada ve Maya ikiz kardeşlerdir. Babaları onların matematik çalıştığını görür ve çalıştıkları konuya şöyle bir göz ucuyla bakar ve aklına bir fikir gelir. Ada ve Maya'nın ne zamandan beri istedikleri bir hikaye kitabı serisini alır ve gelir. Kızlarına da şöyle der: "Şimdi kızlar size iki tane sayı söyleyeceğim, Ada o sayıyı en yakın onluğa, Maya ise ikinci söyleyeceğim sayıyı en yakın yüzlüğe yuvarlayarak tahmin edecek. Ben de ikinize söylediğim sayıları çıkaracağım, siz de yuvarlayarak elde ettiğiniz sayıları çıkaracaksınız. Tahminlerinizden elde ettiğiniz fark gerçek sonuçtan fazlaysa size aldığım hikaye serisini okumaya ilk Maya başlayacak ama tahminlerinizden elde ettiğiniz fark gerçek sonuçtan az ise hikaye serisini okumaya ilk Ada başlayacak. Bu sayede hem öğrendikleriniz pekişecek hem de istediğiniz hikaye kitabı serisine kavuşmuş olacaksınız." Ada ve Maya'nın babasının söylediği ilk sayı 4 032, ikinci sayı ise 5 209 olduğuna göre hikaye kitabı serisini ilk okumaya kim başlamıştır?</p> <p>Ada and Maya are twin sisters. Their father saw that they were studying mathematics, and he looked at the subject that they were studying and had an idea. He got the story book series that Ada and Maya wanted. He says to his daughters: "Now girls, I will tell you two numbers. Ada will predict that number to the nearest ten, and Maya will predict the second number to the nearest hundred. I will subtract both numbers that I told you. You will subtract both numbers that you got by rounding. If the difference is more than the actual result, Maya will start to read the storybook series first. But if the difference you got from your predictions is less than the actual result, Ada will start to read the storybook series first". Since the first number is said by the father is 4032 and the second is 5029, who did start to read the storybook series first?</p>
<p>Solution</p> <p>Ada en yakın onluğa yuvarladığı için söylediği sayı 4 030 Maya en yakın yüzlüğe yuvarladığı için söylediği sayı 5 200 olur. Bu sayıların farkı $5\ 200 - 4\ 030 = 1\ 170$ Gerçek fark $5\ 209 - 4\ 032 = 1\ 177$ Tahminlerinden elde ettikleri fark gerçek sonuçtan az olduğu için hikaye serisini okumaya ilk Ada başlayacak.</p>

Figure 6.

The Second Example for Level 2

subtract the numbers from each other” and stated more clearly which numbers they were. It is understood what the preservice teacher wanted to ask when the solution was examined, and it is seen that she could solve the problem. However, it is not easy to reach this solution when the problem is read. There are deficiencies or mistakes in the preservice teacher’s use of mathematical language, grammar, instructions, and data. Besides, it seems that the problem is far from originality. The preservice teacher’s problem-posing skill was evaluated as Level 2.

In the seventh example (Figure 7), the preservice teacher created a speed-time problem that required them to use formulas.

Problem

Öğretmen Aslı ve Ahmet saat 10:00 telefonda konuşurlar ve saat 17:00'da şehir dışındaki bir kafe'de buluşma kararı kıyırlar. Aslı yola saat 13:00'da çıkar ve evi ile kafe arası 200 km'dir. Ahmet ise yola saat 14:00'da çıkar ve evi ile kafe arası 300 km'dir. Aslı'nın o ki Aslı'nın yolda benzini bitiriyor ve yardım gelene kadar yarım saat geçiyor. Ahmet'inde gittiği yolda kaza olmuş ve trafik tıkanması nedeniyle 45 dk bekliyor. Buna göre bunların hızları ne kadar olmalıdır ki Ahmet yarım saat geçici de 1 saat geçici saatine geç kalması olmasın?

Aslı and Mehmet talk on the phone at 10:00 am and they decide to meet at a café at 17:00 pm. Aslı takes the road at 13:00 pm and the distance between her house and café is 200 km. Ahmet takes the road at 14:00 pm and the distance between his house and café is 300 km. Aslı runs out of gas on the way and half an hour passes until the help arrives. Ahmet waits for 45 minutes due to the traffic depending on an accident on the way. Accordingly, what speed should they have for Ahmet not to be half an hour late and Aslı not to be 1 hour late?

Solution

Aslı'nın hızı için $13:00 - 17:00 \rightarrow 4$ saat yolda ama benzini bitiriyor için 0,5 saat eklenene göre 4,5 saat hareket etmiş ve bu süreçte 200 km yol almış yani hızı $200 = 4,5 \cdot v$
 $44,4 = v$ olmalı

Ahmet'in hızı için $14:00 - 17:50 \rightarrow 3,5$ saat yolda ama trafik nedeniyle 45 dk eklenene göre 2,75 dk yolda hareket halinde $300 = 2,75 \cdot v$
 $110 = v$ olmalıdır

Figure 7.

The Second Example for Level 3

Similar to the previous problem, this problem is complex and long in terms of following directions and understanding the problem. However, the use of mathematical language and data is more appropriate. The incoherencies exist in the expressions. The problem is solvable but of the ordinary type. It is seen that the preservice teacher could solve the problem posed by herself appropriately. In general, there were a few mistakes or deficiencies in terms of criteria so the problem-posing skills of the preservice teacher for this problem were found at Level 3.

In the eighth example (Figure 8), it is seen that the preservice teacher tried to pose a nonroutine problem, including various instructions and the exploration of the relationships between numbers.

Problem

X tane bilye özdeş iki kutuya aşağıdaki kurallara göre dağıtılacaktır.

- Önce 1.kutuya 1, sonra 2.kutuya 2, sonra 1.kutuya 3, sonrasında 2.kutuya 4, bilye atılacaktır.
- En son kalan bilyelerin sayısı kurala uymadığı takdirde sıra hangi kutudaysa kalan bilyelerin hepsi bu kutuya atılacaktır.

Bu işlem sonunda 1.kutuya son kalan bilyeler atılmış ve 1.Kutuda toplam 175 bilye bulunduğuna göre x kaçtır ?

x balls will be placed in two identical boxes according to the following rules.

- First, 1 into the 1st box, then 2 into the 2nd box, then 3 into the 1st box, then 4 into the 2nd box,
- If the number of the last remaining balls does not comply with the rule, all the remaining balls will be thrown into the next box.

As a result of this operation, the last remaining balls are thrown into the first box and there are 175 balls in it, what is x?

Solution

Son kalan a tane bilye 1.kutuya atılmış ise kutulara atılan bilye sayıları sırasıyla

1. kutu	2. kutu	
1	2	
3	4	
5	6	
⋮	⋮	
2n-1	2n	
+		şekindedir:
a		

Buradan

$$1+3+5+\dots+(2n-1)+a=175$$

$$n^2+a=175$$

$$13^2+6=175 \text{ olacağı için } n=13 \vee a=6 \text{ olduğu görülür}$$

$$x=1+2+3+\dots+(2n)+a$$

$$x=1+2+3+\dots+26+6$$

$$x=357$$

Figure 8.

The Second Example for Level 4

For this problem, the suitability of the use of mathematical language, grammar, and expressions is high. It is seen that there is no incoherency and that the directions and data are clear and adequate to solve the problem. The solution of the preservice teacher is appropriate, and the problem is a different type from the classical. Thus, the problem-posing skills of the preservice teacher were accepted as Level 4.

Discussion, Conclusion and Recommendations

The purpose of this study was to investigate the problem-posing skills of preservice mathematics teachers and the relationship between their problem-posing skills, thinking styles, and academic achievement. The findings of descriptive statistics showed that the preservice middle school mathematics teachers' problem-posing skills were at Level 4, meaning that they were high. It reveals that they are successful

in posing their own problems. Moreover, it was found that they were generally good at using mathematical language; solving their own problems; giving appropriate directions; and indicating adequate data in terms of quality and quantity, although there were some grammatical mistakes or incoherence in problems. It shows that preservice teachers draw attention to pose appropriate and solvable problems. Similarly, some research has stated the capability of students and teachers in posing mathematics problems (Cai, 2003; Cai et al., 2013; Cai & Hwang, 2002; Crespo, 2003; Kar, 2015; Silver & Cai, 1996; Stickles, 2011). Cai (2003) also indicated that as students' grade levels increase, the percentages of their success also increase. In contrast, Korkmaz and Gür (2006) found that the majority of preservice primary and mathematics teachers were not able to pose problems. In this research, the reason for preservice teachers' success in posing problems may be that they were free about what kind of problems to pose. Besides, the education they received during the problem-solving course may also have positively affected their problem-posing success. However, the originality of the problems was low. Some studies also emphasized the scarcity of original problems that were posed by students (Özgen et al., 2019; Tertemiz & Sulak, 2013) and preservice teachers (Korkmaz & Gür, 2006). The findings show that preservice teachers have mostly tended to pose routine problems that are similar to those they encounter or those they are sure to solve. Silber and Cai (2017) found that most of the preservice teachers posed solvable problems. In the literature, it was concluded that students who successfully posed problems consider possible solutions for the problems while posing them (Cai, 1998; Silver & Cai, 1996). However, there were also preservice teachers who could not solve the problem they created or posed problems that could not be solved, even if they were a small amount. Considering the strong relationship between problem solving and problem-posing (Cai, 2003), the increase in problem-solving success can be provided by the integration of problem-posing activities into the learning-teaching process (Dickerson, 1999; Özgen et al., 2017). It was also observed that although some problems cannot be solved owing to the lack of data and expression, preservice teachers solved them by completing the missing data and directions. Moreover, the tendency of writing long problems to provide originality draws attention but it is seen that this attempt may cause complexity and decrease the meaningfulness of the problems.

The findings of correlation analyses show that there is no significant relationship between problem-posing skills and the thinking styles of preservice teachers. It can be said that the problems posed by preservice teachers are not significantly affected by their thinking styles. This may be due to the fact that preservice teachers do not want to spend much time thinking about how to pose their original problems, they more rely on the problem styles they encounter in the educational system and they

doubt the adequacy of the problems they will create themselves. It was also found that there is no significant relationship between academic achievement and problem-posing skills of preservice teachers. It reveals that the academic achievement of preservice teachers does not have a significant role in the problems that they posed. In contrast to the results of this study, some studies found that students with high academic achievement and mathematical success are more successful in problem-posing (Akay & Boz, 2009; Dickerson, 1999; Nicolaou & Philippou, 2007; Özgen et al., 2017). The reason for this contradiction may be that most of these studies were conducted at lower-grade levels. On the contrary, the findings show that academic achievement was significantly, positively, and moderately related to executive, hierarchical, and internal thinking styles, whereas it was significantly, negatively, and moderately related to external thinking style. It can be said that the preferences that are concerned with following instructions, implementing guidelines, taking priority, considering hierarchy, and working on tasks on one's own may contribute to the academic achievement of preservice teachers. The preference for working on tasks collaboratively may have negative effects on academic achievement. The results of the studies on the relationship between thinking styles and academic achievement revealed that thinking styles that require conformity (conservative), respect for authority (executive), and a sense of order (hierarchical) were positively related to academic achievement (Bernardo et al, 2002; Zhang, 2001a; Zhang, 2001b; Zhang & Stemberg, 1998). Furthermore, similar to the findings of this study, some studies found that a preference for working individually (internal style) was positively correlated with academic achievement, whereas a preference for working in groups (external style) was negatively associated with academic achievement (Zhang, 2001a; Zhang, 2001b; Zhang & Stemberg, 1998). Similarly, Cano-García and Hughes (2000) stated that students' academic achievement and thinking styles were not independent and that students who prefer to work individually (internal) and who have adherence to existing rules and procedures (executive) had higher academic achievement. Differently, Zhang (2002c) found that the thinking styles that correlated significantly with achievement were liberal, global, and conservative. Buluş (2006) revealed that anarchic and conservative thinking styles were significantly but negatively correlated with achievement.

Limitations

Academic achievement and the factors that influence it are one of the essential topics in education. This study focused on the relationship between thinking styles and academic achievement, and their contributions were indicated. However, the next step should be focusing on how and why particular thinking styles influence academic achievement and how it differs depending on the

subject matters. Besides, the effects of teachers' thinking styles on students' academic achievement can also be investigated to gain more insight into the teaching and learning process. In contrast to the studies in the literature, it was found that there was no relationship between problem-posing skills and academic achievement. This contradiction reveals the need for more research with preservice teachers on problem-posing. Participation in problem-solving courses is thought to be effective for preservice teachers to have high problem-posing skills. This situation reveals the importance of such lessons. It is recommended to add a problem-posing course as well as a problem-solving course in the undergraduate program. The results showed that preservice teachers have a tendency to pose routine problems that they are familiar with. It is believed that this kind of course will be useful to increase the quality of the problems they will pose and develop their abilities to pose nonroutine problems. The participants were limited, and no intervention was performed in this study. Future studies with experimental research designs, long durations, different variables, and more participants need to be conducted.

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